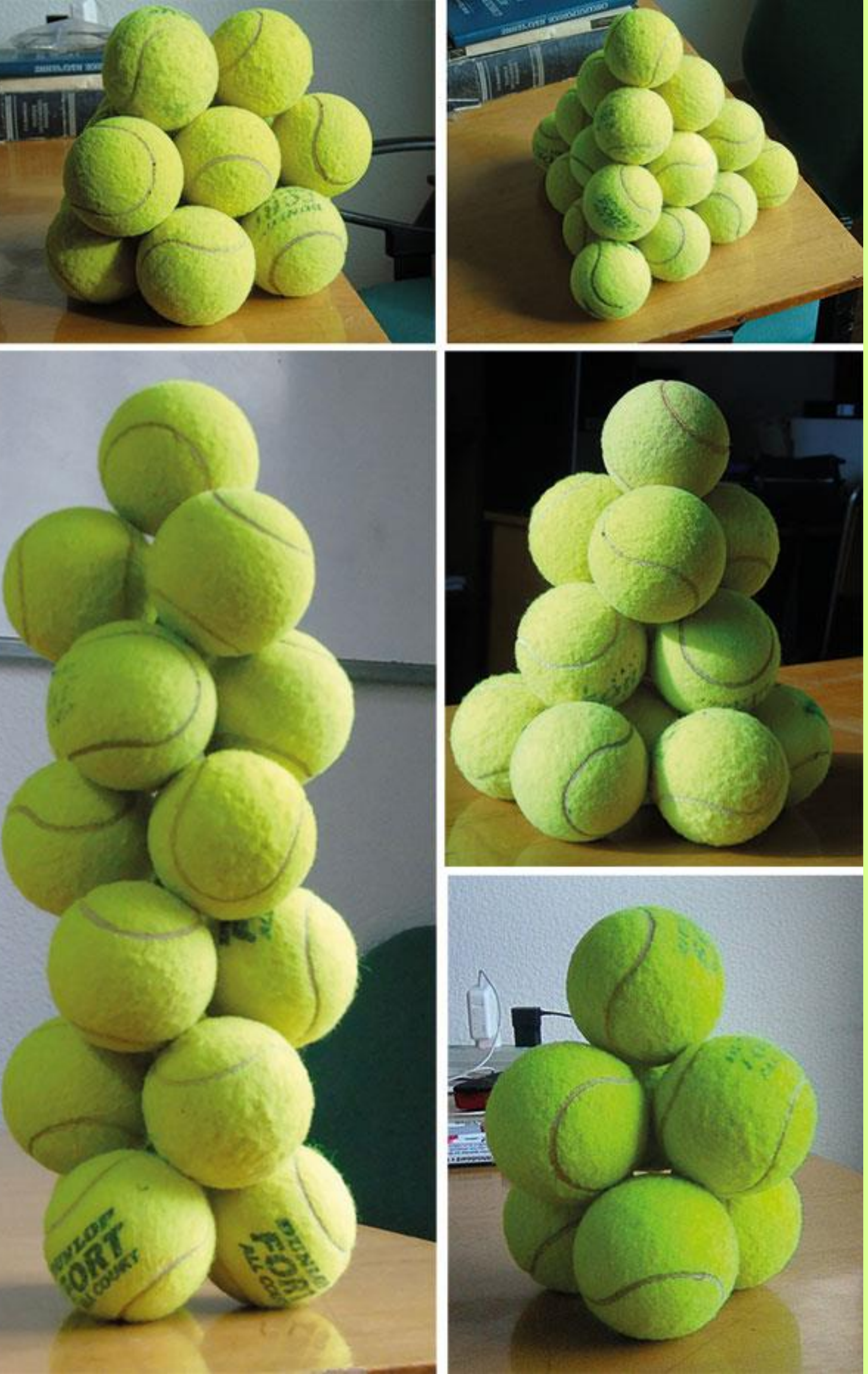

6. Tennis Ball Tower

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Co-funded by the
Erasmus+ Programme
of the European Union

IYPT
International Young Physicists' Tournament

Assignment

Build a tower by stacking tennis balls using three balls per layer and a single ball on top. Investigate the structural limits and the stability of such a tower. How does the situation change when more than three balls per each layer and a suitable number of balls on the top layer are used?

Key word: FRICTION

Analysis of assignment

- Tower of tennis balls with three balls in each level with single ball on the top
- Stability - stands still?
- How many levels can be built?
- How many balls may be in each layer?
(inspiration in photos)

What is a tennis ball?

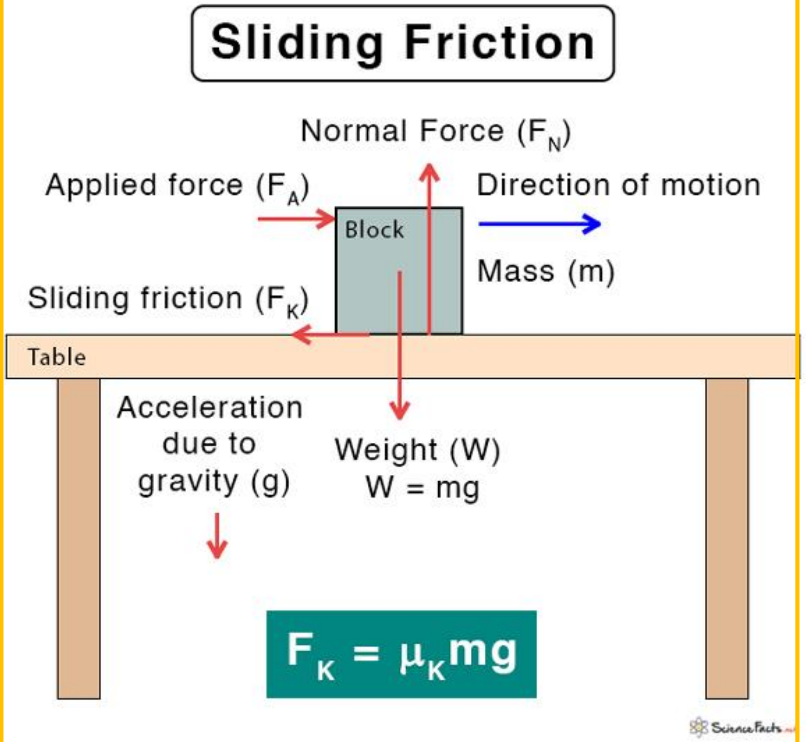
Tennis balls – standardized

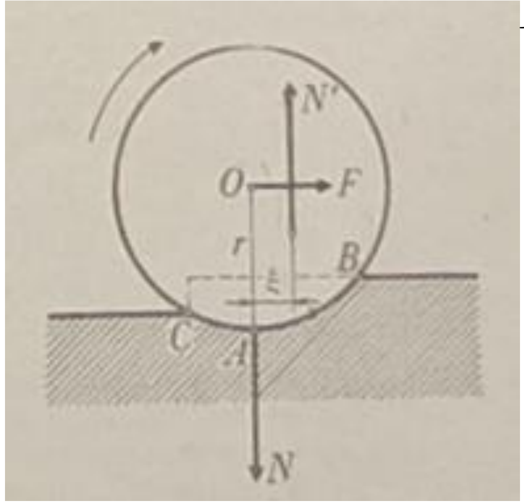
- mass: **56-59,4 g** ;
- diameter: **6,541 – 6,858 cm** ;
- friction coefficient (according to use of the ball):

  0,49-0,7 (hard court) - 0,6 (grass) - 0,8 (clay) ;
- manufacturer quality - price
  (different features using different brands?)
- new vs. used ball
  (change in mass, friction coefficient, elasticity...
  What could be important for our experiments?
  Investigate!)
- White dent around the ball – affects stability?

Friction

- There is a distinction between the types of friction:
- Sliding/rolling
- Static/ dynamic
- Sliding friction is larger than rolling
- Static friction is larger than kinetic
- Friction force is defined as dot product of normal force and corresponding friction coefficient

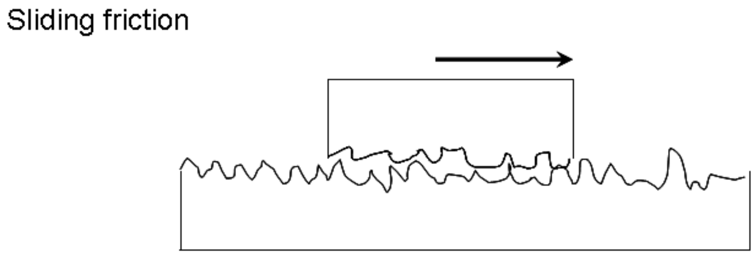


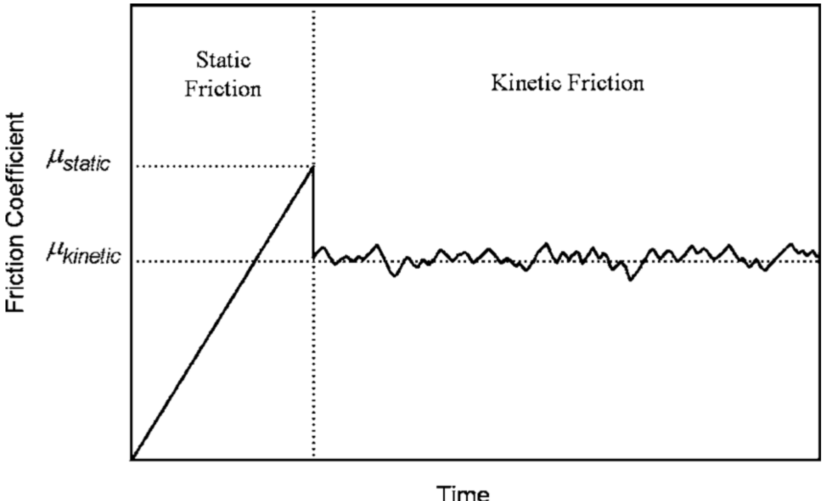


- when moving forward, the ball pushes the surface in front of it
- the movement is inertial:

$N \cdot \xi = F \cdot r$

$\vec{N}$ – normal force, deforms the surface
$\vec{N}'$ – reaction force of the surface
force $\vec{N}'$ is applied in the distance ξ from the axis, creating TORQUE: $\vec{M} = \vec{N}' \cdot \xi$
pair of forces $\vec{N}, \vec{N}'$ rolls the ball around A





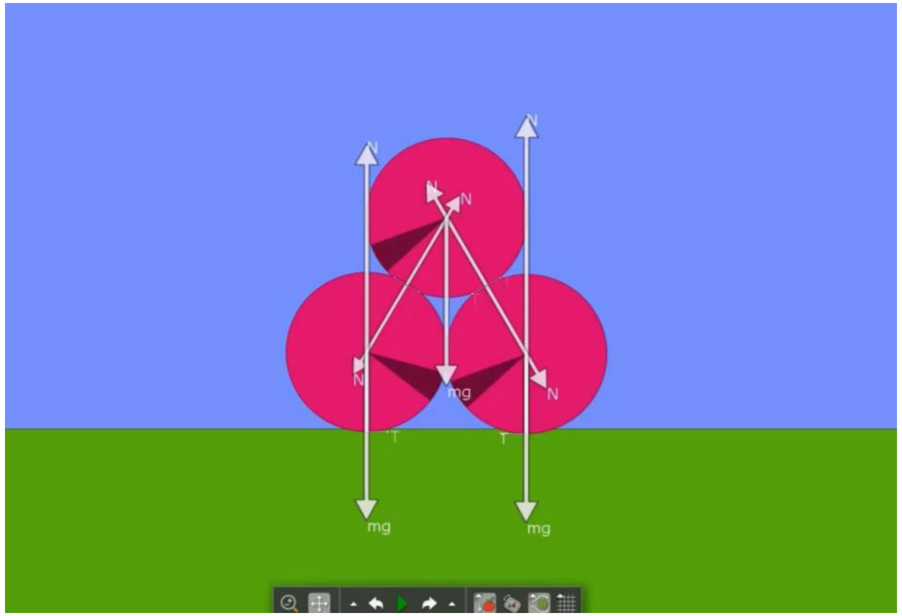
Comparison of different friction coefficients in 2D
How does friction affect stability?

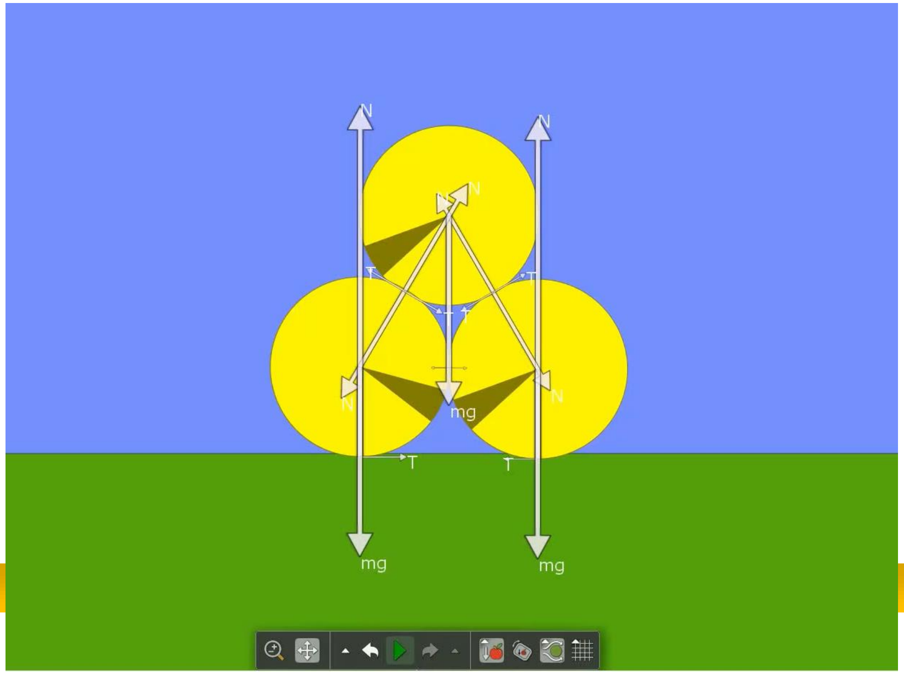
- Top tower (pink):
- Friction coefficient 0,15

- Bottom tower (yellow):
- Friction coefficient 0,5

- Higher friction coefficient prevented the whole tower from collapsing
- [Tool here: http://www.algodoo.com/](http://www.algodoo.com/)

Algodoo simulation:



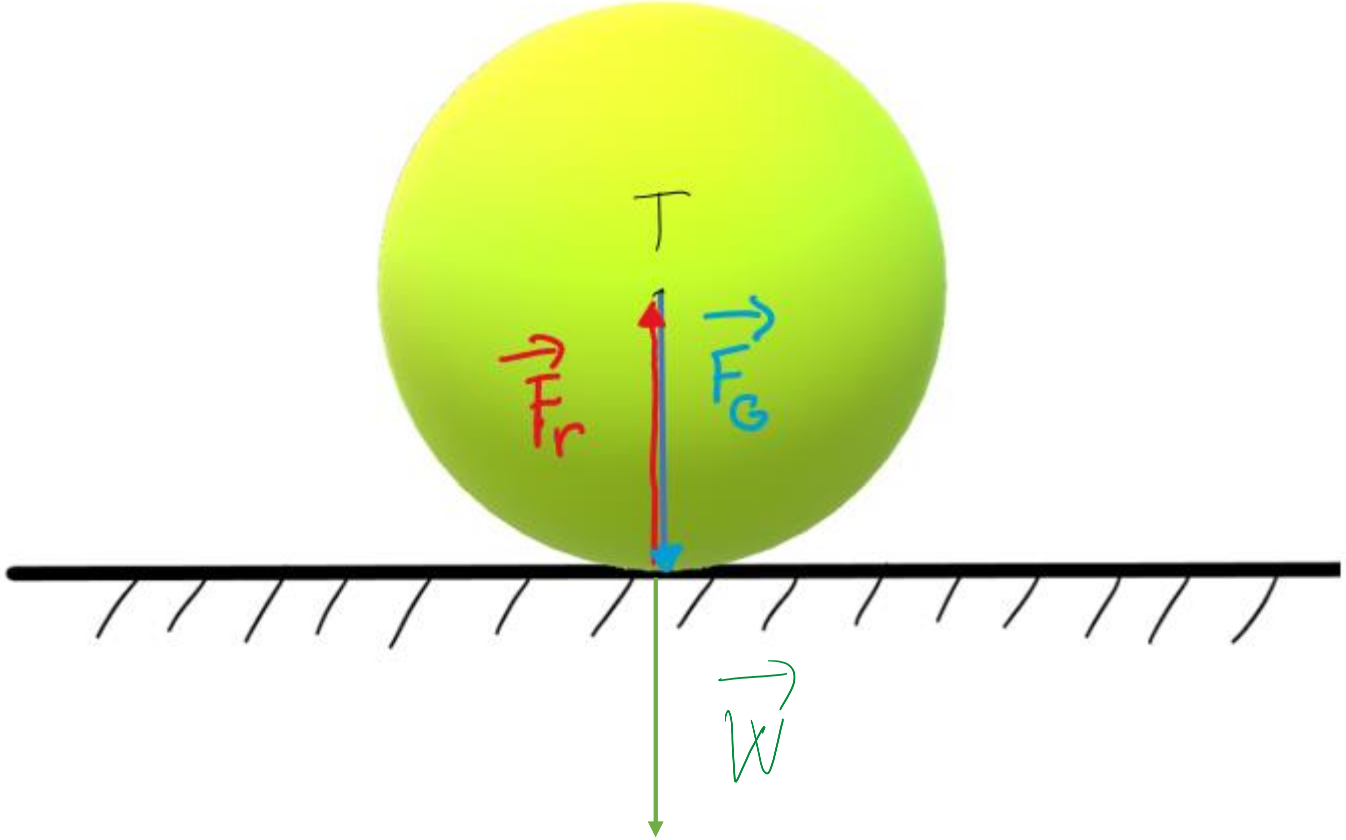


What affects the stability of tower?

- **Stability – amount of work needed to change the stable position of a system (equilibrium) into an unstable one**
  Could it be defined in a different way?
- Stability depends on the position of centre of gravity
- The tower remains in a stable position as long as the centre of gravity is in rest (1st Newton´s Law)
- When the tower is falling apart, the balls not only slide, but they perform rotary motion caused by torque
- The tower holds together by **friction** (it compensates the torques!)

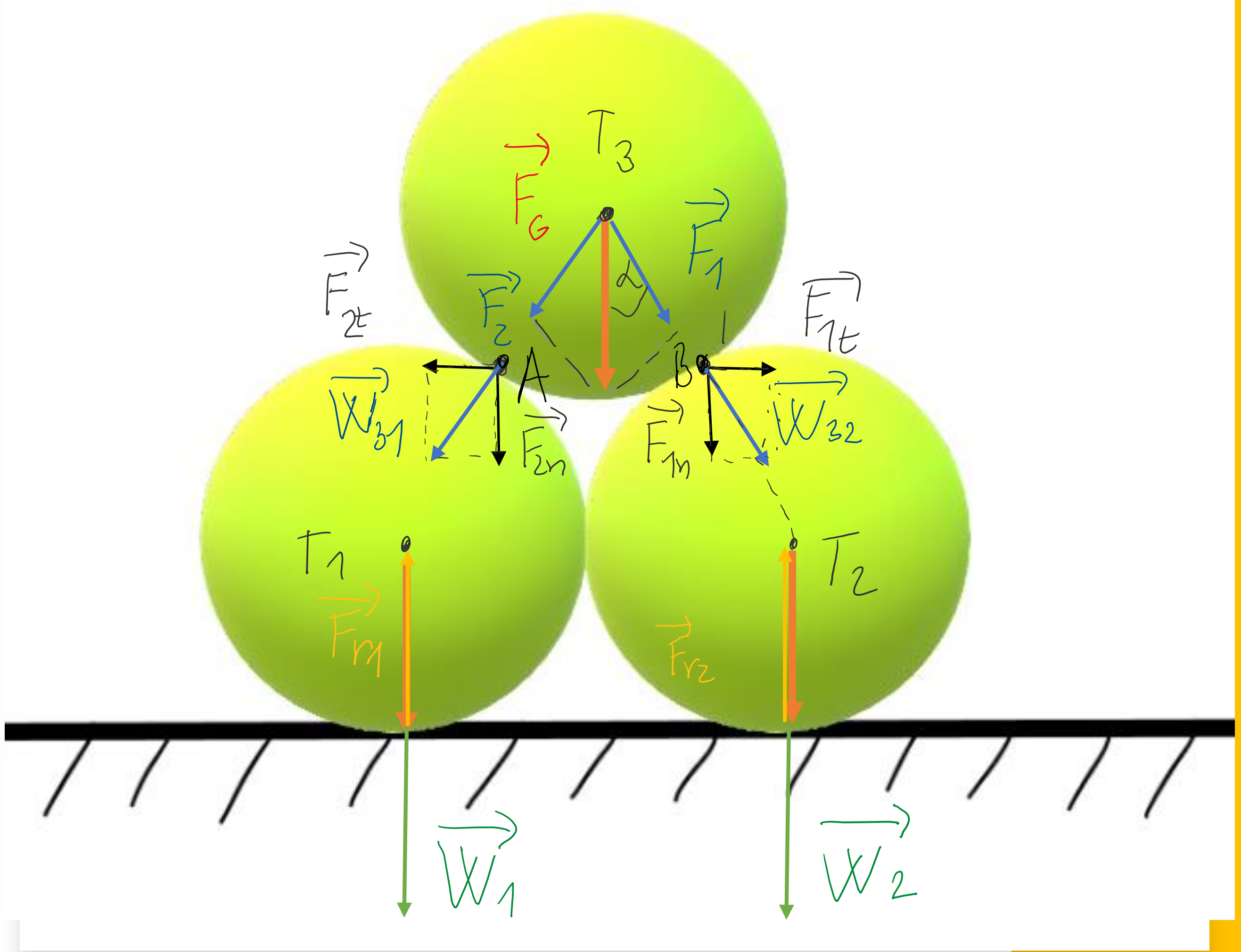
Analysis of forces – single ball on a pad

T – centre of mass
F_G – gravitational force on the ball
F_r – force of reaction of the pad
(3rd Newton´s law)
W - weight



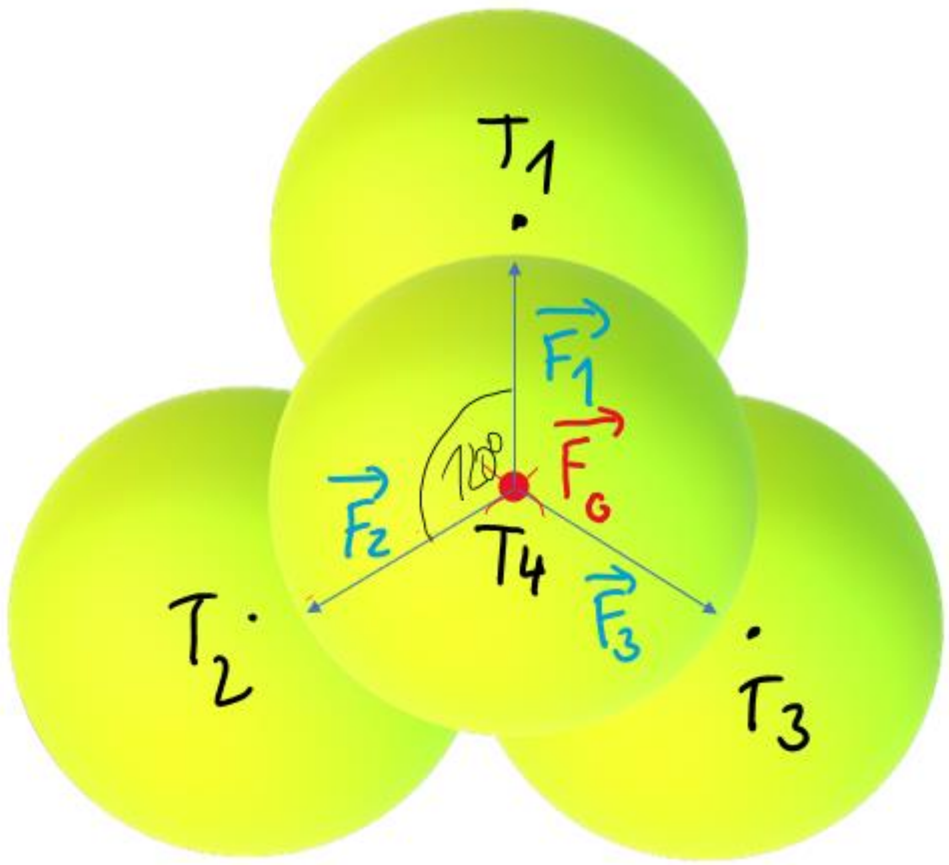
Simplified model: Analysis of forces – 3 stacked balls in 2D

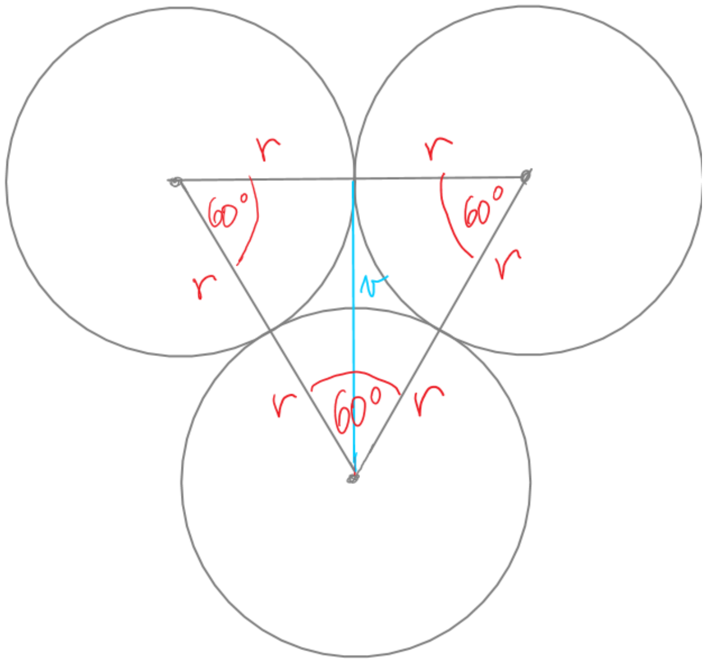
- T_1, T_2, T_3 – centres of gravity of ball 1, 2, 3
- F_G – gravitational forces (red)
- $F_{1,2}$ – resolution of the gravitational force of the ball on the top
- W (indexed) – weight (tiaž)
- F_r – reaction force of the pad on the balls on the bottom (yellow)
- $F_{1,2t/n}$ –decomposition of weight

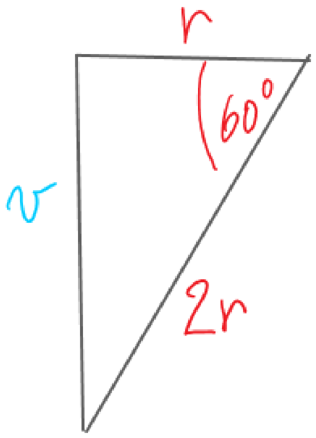


Analysis of forces – 4 stacked balls in 3D – top view

- T_1, T_2, T_3, T_4 – centres of gravity of ball 1, 2, 3, 4
- F_G – gravitational force of the topmost ball
- F_1-F_3 – decomposition of FG into the directions of T_1, T_2, T_3
- Very difficult to draw correctly



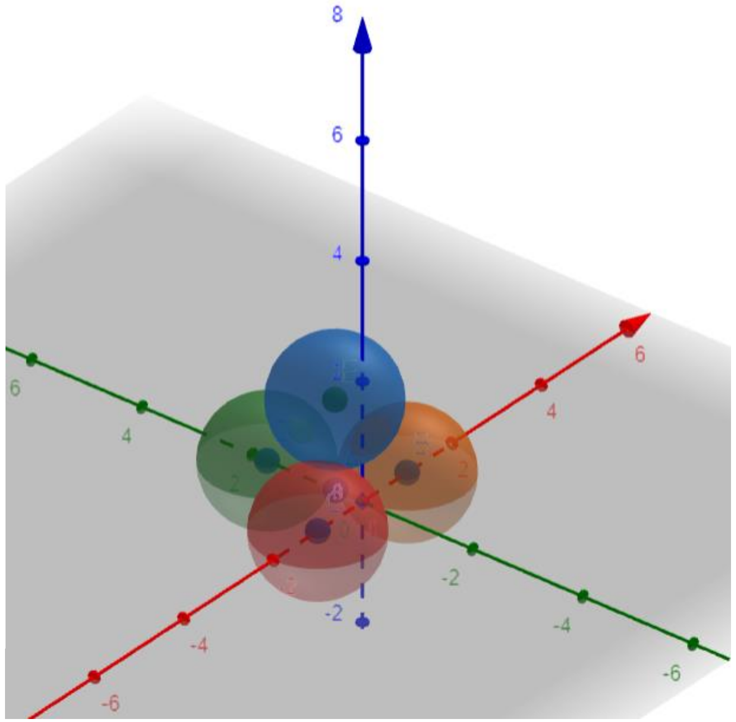




$$tg\,60^\circ = \frac{v}{r}$$

$$v = r \cdot tg\,60^\circ$$

$$v = \sqrt{3} \cdot r$$



Analytical approach to the model of the tower

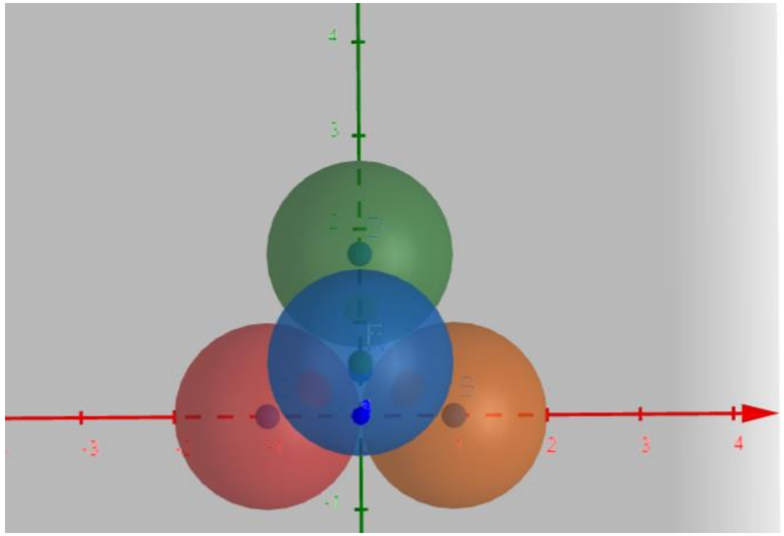
- We need to know the distance of the radii (v) to draw the balls into Geogebra – better visualisation

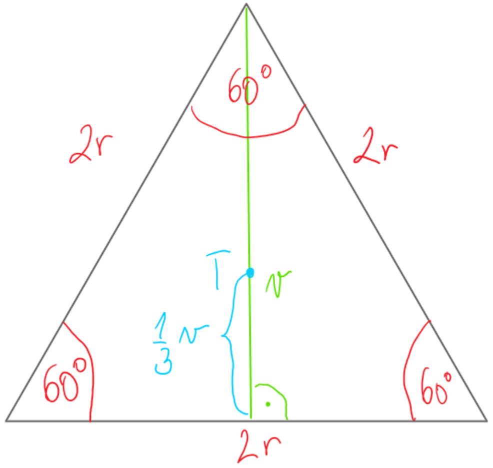
  model here: https://www.geogebra.org/3d/bhnrwmsj

- Trigonometry

Analytical approach to the model of the tower

- Getting v from the previous slide is not enough (it only decides about the coordinates in xy plane of the first floor of the tower)
- How to find the coordinates of the third ball set on the top of the first floor?
- The x coordinate will be 0, y will be 1/3 of the v that we already know
- The z coordinate will be the r+y coordinate (due to symmetry)

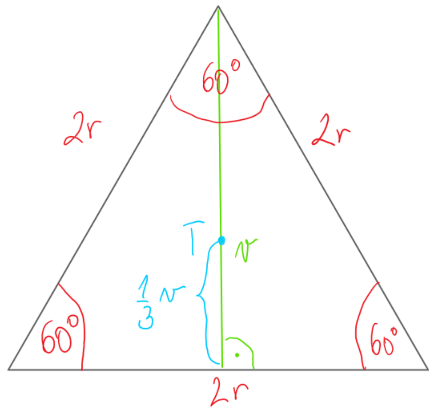


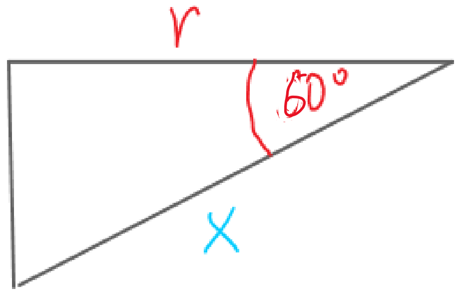


Analytic approach to the problem of tower

- Once we have the coordinates, we can clearly see the „pyramid“ of forces, where the side is looking like this:
- The x distance is the radial distance - the weight of the top ball decomposes into three equal parts with the ratio x

- [Model here: https://www.geogebra.org/3d/jzkxczsf](https://www.geogebra.org/3d/jzkxczsf)





$$\cos 60° = \frac{r}{x}$$

$$x = \frac{r}{\cos 60°}$$

$$x = 2r$$

$$\boxed{F_{1,2,3} = \frac{F_G}{3} \cdot 2r}$$

The centre of gravity of system

$A = (3{,}3 ; 0 ; 0)$

$B = (-3{,}3 ; 0 ; 0)$

$C = (0, \sqrt{3} \cdot 3{,}3 ; 0)$

$D = (0, \frac{\sqrt{3}}{3} \cdot 3{,}3 ; 1{,}58 \cdot 3{,}3)$

$m = m_A = m_B = m_C = m_D$

$m = 0{,}058 \text{ kg}$

$r = 3{,}3 \text{ cm}$

$$T = \frac{\sum_{i=1}^{4} m_i \vec{r}_i}{M}$$

$$M = \sum_{i=1}^{n} m_i$$

$$T = \frac{Am + Bm + Cm + Dm}{4m} = \frac{\cancel{m}(A+B+C+D)}{4\cancel{m}} = \frac{A+B+C+D}{4} =$$

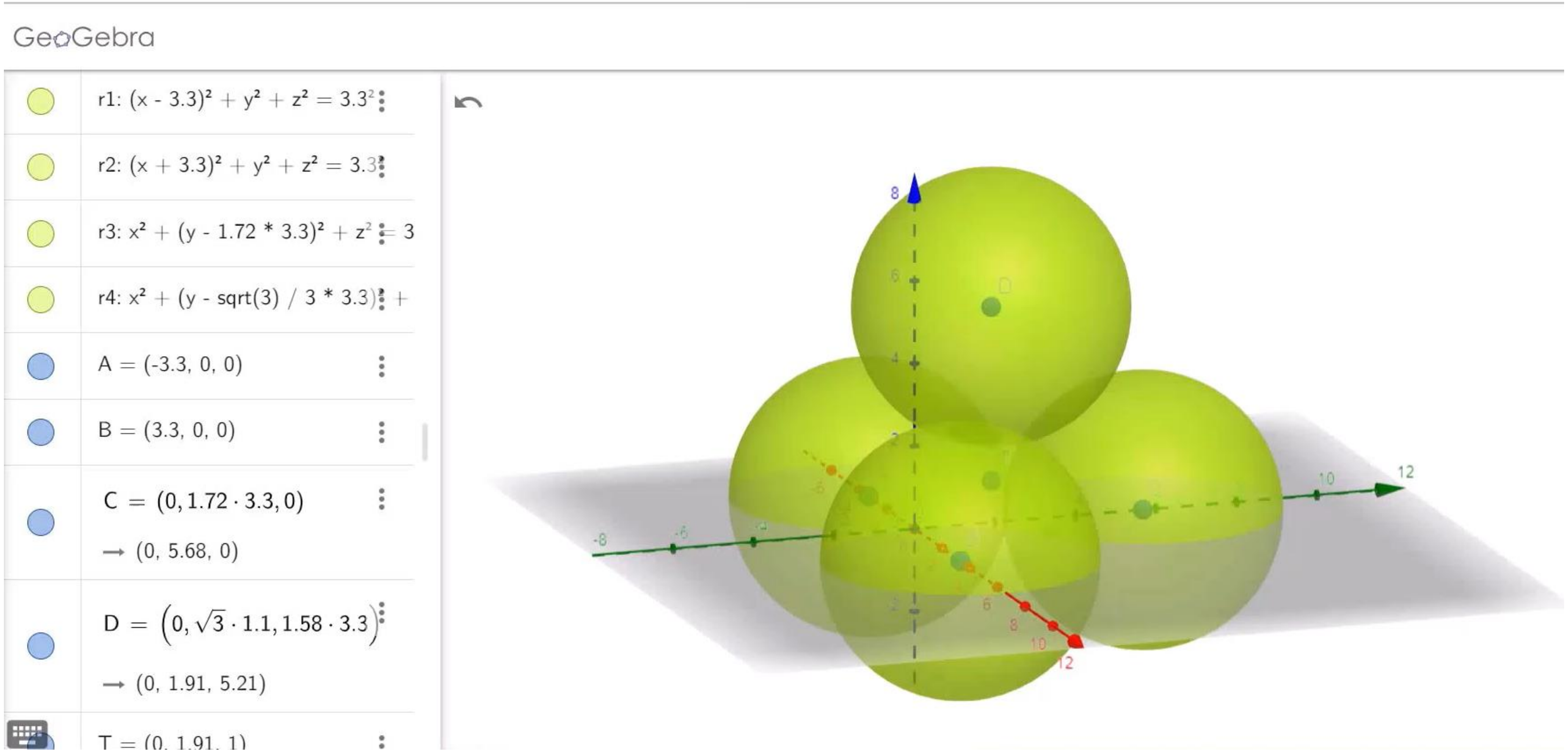
$$= \frac{(3{,}3 - 3{,}3 + 0 + 0 ; 0 + 0 + 3{,}3\sqrt{3} + \frac{\sqrt{3}}{3} 3{,}3 ; 0 + 0 + 0 + 1{,}58 \cdot 3{,}3)}{4} =$$

$$= \frac{(0, 1{,}1 \cdot \sqrt{3} \cdot 4, 5{,}21)}{4} = (0; 1{,}1 \cdot \sqrt{3}; 1{,}3)$$

- If we approximate each ball in the tower with a point mass, we may calculate the position of centre of gravity

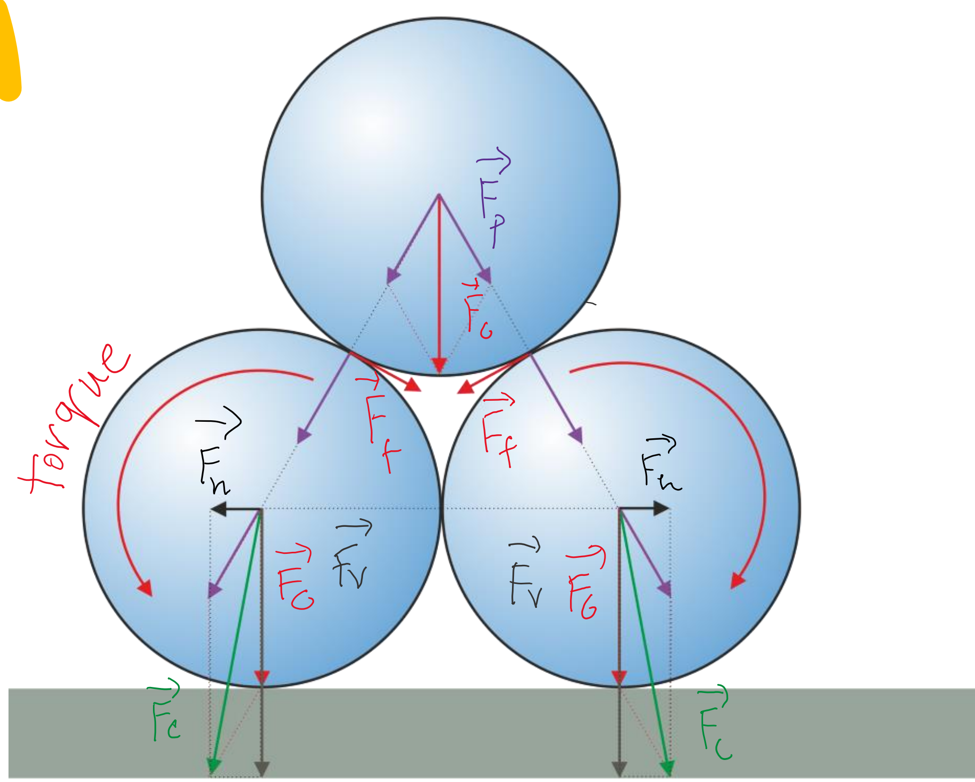
[3D model: https://www.geogebra.org/3d/w2788zms](https://www.geogebra.org/3d/w2788zms)

3D representation of 2 storey tennis ball tower – with centre of gravity



When will the tower collapse?

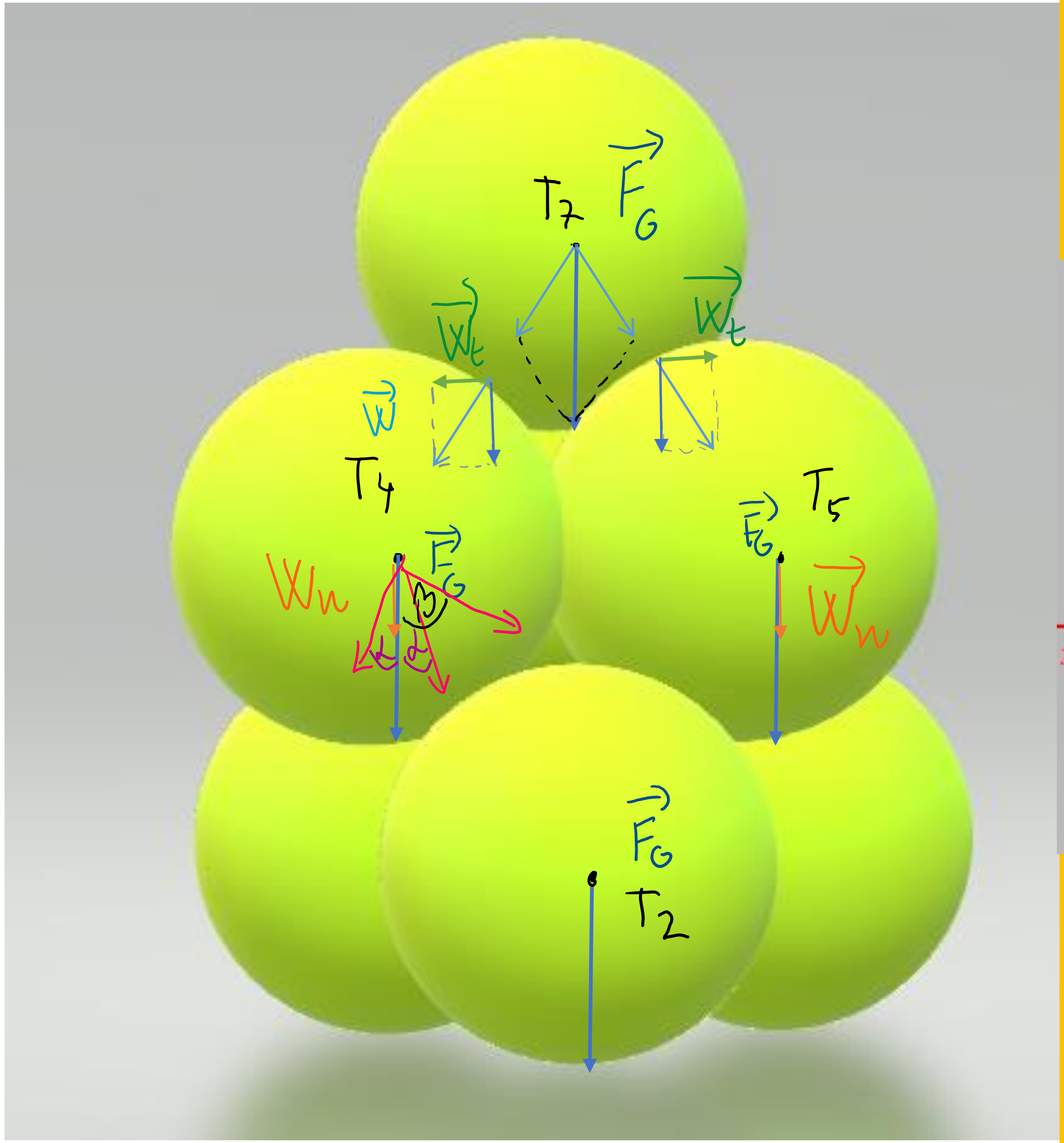
- The wieght of the top ball is decomposed into 2 directions (partial force F_P – purple)
- By shifting the partial force into the centre of gravity of the bottom balls, composing it with the gravitational force F_G into F_C, and decomposing it into vertical F_V and horizontal F_h vectors, we get force F_h which gives torque (with arm of force = radius of ball) in the drawn direction
- Between the top ball and bottom balls, there is friction force F_f perpendicular to radius, aiming in between the balls
- If the torque is bigger than friction force, the bottom balls roll out and the tower collapses
- Note: friction force between balls of each layer may be omitted



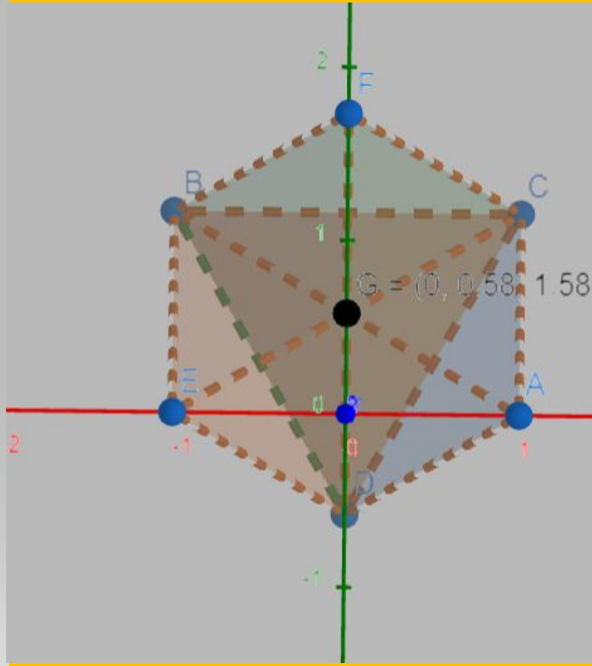
What happens if we have three layers?

[3 storey tower point mass – GeoGebra]

Try to calculate the position of the centre of gravity of such system
Analyze the forces
The GeoGebra model is for unitary radii



Top view from Geogebra - symmetry



It´s up to you: What happens if we have n layers?

- The pattern of decomposition of forces repeats
- What role does the ball on the top play?
- Is it (force-wise) much different if there is 1 or 8 layers between the bottom and top balls?
- How does number of layers affect friction (normal forces)?
- What happens to the centre of gravity as we add more layers? How does that affect the stability? Does deformation play role?

Which parameters can be investigated?

- Surface on which we build the tower – different friction coeffiecients between the balls and the pad
- Distance between the balls in the layers (do they have to touch?)
- CAUTION!
  When building tennis ball tower, ensure that you have a water-level so that all balls in the bottom layer have the same potential energy
- Tennis balls – new/used, different manufacturers, friction coefficients
- Number of balls in each layer if we investigate the second part of the assignment

How to determine the number of balls in a layer?

- By trying ☺
- Physics – the balls should interlock, so there should be enough space between the balls to fit top balls in the gaps
- Try even numbers
- **How does the situation change when more than three balls per each layer and a suitable number of balls on the top layer are used?**
- Try to build a pyramid – and physics starts again (analysis of forces, stability...)
- Play with it!

Do you have any questions?

Thank you for your attention!

- Literature:

- https://physicsworld.com/a/physicist-creates-remarkable-tennis-ball-pyramids-including-one-made-from-46-balls/
- https://stemfellowship.org/iypt-references/problem6/
- https://www.dailymail.co.uk/sciencetech/article-7061931/Physicist-creates-sculptures-tennis-balls-using-FRICTION-together.html
- https://www.gypt.org/aufgaben/06-tennis-ball-tower.html
- https://twu.tennis-warehouse.com/learning_center/balltesting.php